The investigations showed that the gas-discharge ion source under consideration has high energy characteristics and is effective for obtaining a jet of rarefied synthesized plasma with parameters satisfying conditions simulating the flight of aircraft in the ionosphere. The introduction of a flow of synthesized plasma into a longitudinal magnetic field, modeling the magnetic field of the earth, has been realized, and the gradients of the potentials along the jet of plasma in this field have been determined.

The authors express their thanks to G. L. Grodzovskii for his interest in the work and for his fruitful discussion of the questions touched on in the article.

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NATURE OF SIGNAL EXTRACTED FROM A TOTAL ELECTROMAGNETIC PULSE

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§1. It is well known [1] that gamma rays passing through air excite electromagnetic fields by currents produced by Compton electrons that are formed during the interaction of gamma quanta with the atoms and molecules of air. In the idealized case of an isotropic source and a homogeneous medium this field is present only in the current zone; however, in actuality there is always some asymmetry present which leads to radiation of electromagnetic waves.

In [1, 2] the fields are computed for the current zone as well as the wave zone for a gamma-ray pulse that damps exponentially in time; here the nature and origin of the spatial asymmetry in the distribution of the radiating currents were not specified. The model problem of fields excited by a pulsed gamma-ray source, located at the plane boundary of the half-space formed by an ideal conductor and homogeneous air, is solved in [3]. The problem for an isotropic source in inhomogeneous air is investigated in [4] without considering the effect of the underlying surface. In [1-4] the air density is normal or close to it. The

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pulses of the radiated field obtained in [2] and [3] are shown in Fig. 1 (curve 1 is computed for parameter $N = 1.87 \cdot 10^{22}$, curve 2 for $N = 2 \cdot 10^{24}$, and curve 3 for $N = 2 \cdot 10^{25}$).

The vertical component of the electric field, recorded at a distance of 44.6 km from the source and published in [5], is shown in Fig. 2.

A noteworthy fact is the presence of qualitative and quantitative differences between the experimental and theoretical results. The disagreement between the lengths of the signal is most characteristic. The theoretically computed pulse has characteristic half-periods of a few microseconds and total length of the order of tens of microseconds, whereas the experimentally observed field variations are characterized by periods of the order of tens of microseconds and last for hundreds of microseconds. Similar disagreements are observed also in the ratios of the field amplitudes in different half-periods. For the experimental pulse these ratios are of the order of unity, while for the theoretical pulse they are of the order of 10. It can be shown that practically no modifications (without going beyond the realm of physical possibility) of the radiation mechanisms discussed in [2, 4] would lead to a significant improvement in the agreement between the theory and the experimental results. Actually, the process of formation of the radiation pulse is investigated in [6] from a fairly general point of view (in spite of the fact that a specific case of radiation related to the effect of an external magnetic field is investigated in this work, the qualitative results remain unchanged even in the present case), where it is shown that the total length of the radio pulse is determined by the size of the current zone, which has a magnitude of a few (up to 10) mean free paths of the gamma quanta. This gives tens of microseconds for the total duration of the field, which is also confirmed by more accurate computations. The only possibility of bringing about an agreement between the theory [2, 3] and experiment [5] is to assume that in the radiation of the electromagnetic pulse published in [5] another effect of a different nature made a contribution besides the effect from the currents produced by Compton electrons discussed in [2, 3], so that the total recorded signal is a sum of two signals, one of which cannot have any relation to the excitation mechanism investigated in [2, 3] and others. Let us investigate the possibility of solving the inverse problem of separating the experimentally recorded total signal into two component signals of different natures.

In the present case, the possibility of separating the total signal is determined by the significant difference in the time scales of the signal produced by Compton electron currents and of the other signal. The addition of the signals occurs only in the first 10 μsec, after which the signal from the Compton electron current practically ceases and only the other signal is recorded. Regarding the latter, it must be assumed that it combines additively with the signal from the Compton electron currents, the temporal dependence of this signal is described by an analytical function, and the zero conditions are satisfied at the initial time.

Under these assumptions, the signal produced by the Compton electron current can, in general, be separated by choosing any point on the recorded signal within the time interval, where the signal from the Compton electron current has already stopped, and taking the derivatives of the signal at this point. After this, constructing Taylor series in the neighborhood of this point, the signal, which is not related to Compton electron current, can be computed even in the region where it is added to the signal produced by the Compton electron currents. Subtracting this separated signal from the total signal, one can determine the signal produced by the Compton electron currents. The practical difficulties in realizing this program considerably reduce its potentialities. First of all, we note that at an arbitrary point it is almost impossible to determine even the third derivative of the function; therefore, only the coefficients of the first three terms can be determined in the entire Taylor series. Furthermore, the lack of information on the behavior of the coeffients with the increase of the term number makes it impossible to get an a priori estimate of the region of convergence of the Taylor series. These and a number of other less essential difficulties necessitate the use of a compromise method that does not pretend to have high accuracy and rigor, but helps one to obtain reliable qualitative results. This method is based on choosing the initial point for approximating the additional (to the signal produced by the Compton electron currents) signal by a finite number of terms of the Taylor series. The choice of the required number of terms of the Taylor series is made from the consideration that for analytical functions the convergence of the Taylor series or the specified accuracy of approximation of the function by a finite number of terms of this series occurs within a circle of a given radius in the complex plane of the argument of the function. Therefore, if a certain number of terms of the series are sufficient for the approximation (with the specified accuracy) of the function in the interval $(0, a)$ of variation of the real argument, then it can be assumed that the same number of terms of the series are sufficient also in the interval $(-a, 0)$ of variation of the argument. The requirement of the minimum number of terms of the series forces one to choose the initial point as close as possible to the instant of cessation of the signal from the Compton electron currents. If the inflection point of the signal is taken as the initial point, then after determining the zero- and first-order terms it can be asserted that the second term is equal to zero, and main attention should be given to the determination of the next terms.

We now turn to the analysis of the signal presented in [5].

A priori information about the signal produced by the currents formed by Compton electrons [2-4] permits the assumption that it ceases somewhat earlier than the first zero crossing of the recorded signal ($t_1 \approx 17$ μsec). Thus, the desired point must lie at the inflection point of the signal lying close to the instant of this crossing. We note that the shape of the total signal is such that its antisymmetric (with respect to the chosen point) part is larger than the symmetric part. This fact permits the assumption that besides the zero- and first-order terms (the second-order term is zero) in the expansion, the third-order term plays an important role, and the fourth-order term already has the role of a correction term. Retaining the first five terms we find that the first three (zero, first, and second) are determined with certainty and the remaining two (third and fourth) should be chosen, first, from the condition of vanishing of the approximating expression at the start of the signal and, secondly, from the condition that the approximation of the resultant signal is sufficiently accurate in a time interval (on the increasing side) no smaller than the interval between the start of the signal and the chosen point. The possibility of meeting these requirements with the use of two coefficient-parameters of the third and fourth terms determines the adequacy of the chosen number of the Taylor series for approximating the function in the required interval. In practice, this process requires several trials; the direction of change of the coefficients is determined purely intuitively and it is hard to put the entire procedure in an algorithm. In the case under investigation for the signal shown in [5] and in Fig. 2, point A (see Fig. 2) is chosen as the initial point; the form of the signal added to the signal from the Compton electron currents is interpolated by the expression

$$E = 6.8t - 2.4 \cdot 10^{-2}t^3 + 4 \cdot 10^{-4}t^4,$$

where E is the magnitude of the field (W/m) and t is time reckoned from point A (μsec). The accuracy and the range of approximation can be inferred from a comparison of the approximating polynomial (its graph is shown by the dashed curve in Fig. 2) with the recorded signal for $t > 0$. It is evident that in the required interval the approximation accuracy is no worse than 5%.

According to the discussion presented above, the difference between the recorded and the approximating curves is the signal produced by the currents formed by Compton electrons. This difference is shown in Fig. 1 by the dashed curve (the amplitude and time scales in Fig. 2 coincide with those in Fig. 1; the theoretical curves from [2, 3] are normalized to the same amplitude of the first half). It is evident that the signal separated out from the experimental record agrees quite well in shape and duration with the computed fields radiated by the Compton electron currents (for example, see [3]). A somewhat larger disagreement is observed with the results of [2] with respect to the form of the temporal dependences. To some extent they can be accounted for by the difference in the source intensities; besides, it is probable that the results of [2] are not entirely correct, since in the pulses given in [2] the time integrals of the field do not vanish, whereas it can be shown that in the problem analyzed in [2] they must necessarily be equal to zero.

The form of the pulses presented in [4] are significantly different from the form of the pulse extracted from the experimental curve. This may be an indirect confirmation of the fact that the inhomogeneity of the air density with height is not important in the excitation of fields by a source in air of normal density.

§2. We now turn to the question regarding the nature of the signal that is not associated with the Compton electron currents; as seen from Fig. 2, this signal has a field amplitude ≈40 W/m at a distance of ≈50 km and the time scale of its field changes is ≈20 μsec for a duration of ≈100 μsec. Considering the period of existence and the characteristic time of the field variations, it is natural to associate it with the evolution of a thermal wave and its transformation into a shock wave, since these processes occur at the instants when the signal is radiated and are characterized by the same time scales.

The problem of transformation of a thermal wave into a shock wave is investigated in [7]; to our knowledge, the perturbations of the magnetic field at these instants have not been investigated earlier. Of the studies on closely related topics we mention [8] and [9]; however, in [8] the interaction of weak shock waves with the magnetic field is investigated only in the "quasistatic" approximation; in [9], the object of primary concern was the interaction of the high-temperature region of the explosion with the magnetic field at later stages of development of the explosion in vacuum.

The object of the following discussion is to investigate the effects of interaction of the thermal wave of a violent explosion, transformed into a shock wave, with the magnetic field and to study the development of these effects outside the thermal wave for explaining the observed (see above) magnitudes of the radiated field. As the basis of a quantitative discussion we take Maxwell's equations

$$\mathrm{rot}\,\mathbf{B} = (1/c)\partial\mathbf{E}/\partial t + (4\pi/c)\mathbf{j};\ \mathrm{rot}\,\mathbf{E} = -(1/c)\partial\mathbf{B}/\partial t, \tag{2.1}$$

supplemented by the constitutive equation for the currents

$$\mathbf{j} = \sigma\{\mathbf{E} + (1/c)[\mathbf{v}\times\mathbf{B}]\}. \tag{2.2}$$

Since within the wave the characteristic time σ^{-1}($\sim10^{-12}$-10^{-18} sec) and dimension $c\sigma^{-1}$($\sim10^{-2}$-10^{-8} cm), determined by the conductivity of the gas σ, are much smaller than any temporal and spatial hydrodynamic scales, it is admissible to use the infinite conductivity approximation

$$\mathbf{E} = -(1/c)[\mathbf{v}\times\mathbf{B}];$$

this permits one to obtain from system (2.1) an equation containing only the magnetic field $\mathbf{B}$:

$$\partial\mathbf{B}/\partial t = \mathrm{rot}\,[\mathbf{v}\times\mathbf{B}]. \tag{2.3}$$

The properties of the solutions of this equation are well known (on the effect of "frozen-in" character of the field see, for instance, [10]); however, for the subsequent discussion it is advisable to reformulate the known results as applied to the specific case of thermal wave.

We first consider a plane velocity wave $v = v(x - Dt)$ propagating along the x-axis perpendicular to the external homogeneous field $\mathbf{B}_0$ directed along the z axis. The only nonzero component of the magnetic field B_z satisfies the equation

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial x}(vB) = 0,$$

whose general solution is

$$B = F[\zeta - f(\xi)]/(1 - v/D),\ \xi = x - Dt,\ \zeta = x + Dt,$$

where F is an arbirrary function, $f(\zeta)$ is the solution of the equation

$$\partial f/\partial\xi = [v(\xi) + D]/[v(\xi) - D].$$

If the wave occupies a finite region (along the x axis), i.e., if $v \to 0$ for $\xi \to \pm\infty$, then we have

$$B = B_0[1 - v(\xi)/D]^{-1}. \tag{2.4}$$

If the velocity suffers a discontinuity at some surface $\xi = \xi_0$, then the magnetic field also will have a discontinuity there.

If $v = 0$ before the discontinuity, the magnitude of the jump of the magnetic field is determined by the formula

$$[B] \equiv B(\xi = \xi_0 - 0) - B(\xi = \xi_0 + 0) = v(\xi = \xi_0 - 0)B_0/[D - v(\xi = \xi_0 - 0)].$$

The discontinuity of the magnetic field is caused by current flow along the surface $\xi = \xi_0$ directed along the y axis; the magnitude I of this current is obtained by integrating the first equation of system (2.2) along a contour intersecting the surface:

$$I = \frac{c}{4\pi}[B] = \frac{cB_0}{4\pi}\frac{v(\xi = \xi_0 - 0)}{D - v(\xi = \xi_0 - 0)}. \tag{2.5}$$

The contribution of the displacement currents can be disregarded here due to the factor D/c, representing the contribution of these currents to I.

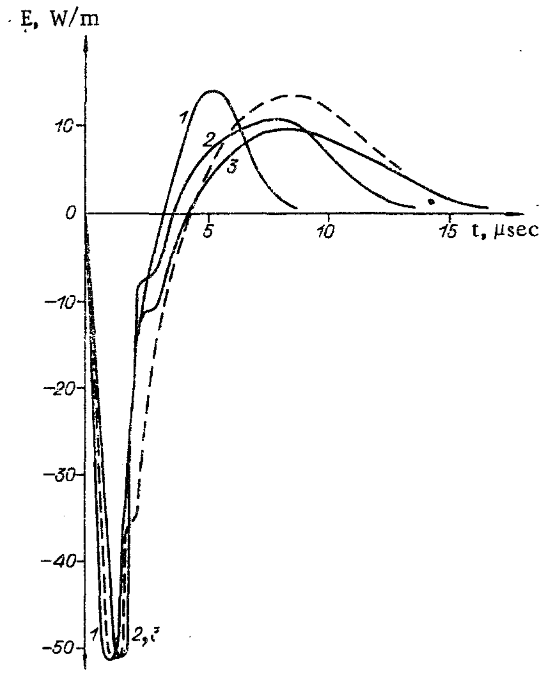

Fig. 1

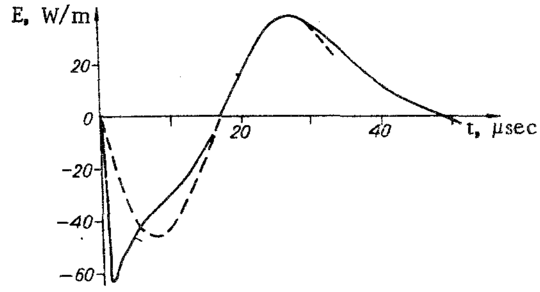

Fig. 2

In connection with results (2.4), (2.5), we note that the field and the currents increase indefinitely as $v \to D$. The equality $v = D$ occurs, for example, during the motion of a gas as a solid body; in real cases, however, the entrapment of the gas that was at rest ahead of the front into the wave must be taken into consideration, so that the inequality $v < D$ always holds, and, therefore, $B > B_0$, i.e., the magnetic field is amplified by the velocity wave.

Passing over to spherical geometry, we discuss the case where the external magnetic field has a single component B_φ in the spherical coordinate system; B_φ depends on the polar angle ϑ, in accordance with the equation

$$B_\varphi(r, \vartheta) = B_0(r) \sin \vartheta.$$

From Eq. (2.3) we then get

$$\frac{\partial B}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rvB) = 0. \tag{2.6}$$

Let us consider the self-similar velocity wave

$$v = V_0(t_0/t)^\alpha f(\xi), \quad \xi \equiv (t_0/t)^\beta (r/r_0),$$

where V_0, t_0, and r_0 are scale constants. We note that for $\alpha + \beta = 1$ the ratio of the gas velocity at any point of the wave (i.e., for fixed ξ) to the velocity of the wave itself [i.e., to the quantity $(dr/dt)_\xi$] remains constant and is equal to

$$v(\xi)/(dr/dt)_\xi = f(\xi)/\lambda\xi, \quad \lambda \equiv \beta r_0/V_0 t_0, \tag{2.7}$$

which makes this formulation analogous to the problem of the plane velocity wave.

For $\beta + \alpha = 1$, Eq. (2.6) has a self-similar solution:

$$B(\xi) = B_0 \exp\left\{-\int_\xi^\infty \frac{d\xi}{\xi} \frac{(f\xi)'}{(\lambda\xi - f)}\right\}.$$

If near the front $\xi = \xi_0$ function f decreases rapidly from f_0 to zero as ξ increases, then the integral in the exponent can be evaluated by taking relatively slowly varying function ξ out of the differentiating symbol and replacing $\lambda\xi$ by $\lambda\xi_0$. After some computations, we find that near $\xi = \xi_0$ the field changes according to the law

$$B(\xi) \approx B_0\lambda\xi_0[\lambda\xi_0 - f(\xi)]^{-1}.$$

Passing on to the limit of discontinuous variation of function $f(\xi)$, we can write

$$B(\xi = \xi_0 - 0) = B_0D/(D - v_0),$$

where we have used the notation $v(\xi = \xi_0 - 0) \equiv V_0$ and $(dr/dt)_{\xi = \xi_0} \equiv D$, taking (2.7) into consideration. Thus, in the spherical case the jump of the magnetic field at the wave front is

$$[B_\varphi] = B_0v_0(D - v_0)^{-1} \sin\vartheta; \; I_\vartheta = (c/4\pi)[B_\varphi], \tag{2.8}$$

which is in complete agreement with (2.5) for the plane case.

The motion of the gas in the inner regions of the thermal wave results in the excitation of currents and fields that do not appear outside the wave, since the high conductivity impedes the propagation of the field. However, the motion of the gas in the zone near the front, caused by the difference in the pressures ahead of and behind the front, results in surface currents flowing along the wave front. These currents may radiate fields including wave signals. The magnitude of these currents can be estimated if we use the law of conservation of mass at the front

$$\rho_1(D - v_0) = \rho_0D,$$

where ρ_1 is the gas density behind the wave front and ρ_0 is the gas density ahead of the front, and replace the ratio $v_0/(D - v_0)$ in (2.8) by ρ_1/ρ_0. The value of this ratio at different instants of time is given in [7], from which we find that, in particular, at the time instants $\approx$40-80 μsec, ρ_1/ρ_0 reaches values of $\approx$8-12. Therefore, the magnitude of the surface currents associated with the effects of the "frozen-in" character of the field

$$I_\vartheta = B_0(c/4\pi)(\rho_1/\rho_0) \sin\vartheta \tag{2.9}$$

is approximately in order of magnitude larger than the magnitude of currents $B_0(c/4\pi)\sin\vartheta$ required for "drawing" out the field from the inner regions of the wave (for example, see [11]). The flow of surface currents (2.9) can be described as the variation of a certain effective dipole moment P of the thermal wave

$$\frac{dP}{dt} = 2\pi R^2 \int_0^\pi I_\vartheta \sin\vartheta d\vartheta = \frac{\pi c}{4} R^2 B_0(R) \frac{\rho_1}{\rho_0}, \tag{2.10}$$

where R is the radius of the wave front. Using the expression for the wave field of an electric dipole, from (2.10) we can determine the radiated signal

$$E = \frac{1}{rc^2}\frac{d^2P}{dt^2} = \frac{\pi}{4rc}\frac{d}{dt}\left\{B_0(R(t))\, R^2(t) \frac{\rho_1(t)}{\rho_0}\right\}. \tag{2.11}$$

A detailed computation of the radiated signal requires detailed information on the distribution of the initial field B_φ and also on the temporal dependences of the gas density and the radius and velocity of the thermal wave front at the initial instants of time. However, a number of estimates can be obtained without detailed computations. Taking $B_0 \approx 10^2$ Oe according to [3] and the values of R and ρ_1/ρ_0 at $\approx$60 μsec equal to $\approx$7 m and $\approx$10 according to [7], and then replacing d/dt by $T^{-1} \approx 10^5$ sec^{-1}, from (2.11) we obtain the following estimate for the electric field at a distance r $\approx$ 50 km:

$$E \sim B_0R^2\rho_1/rcT\rho_0 \approx 10 \text{ V/m},$$

which agrees with the experimentally observed signal (Sec. 1) in order of magnitude.

We note a number of problems which must be resolved for constructing an adequate model of the radiator of the recorded pulse on the basis of the effect discussed above.

The structure of the thermal wave front has a particularly significant effect on the result because it is just the structure of the front that determines the possibility of radiation emanating from the thermal wave through the currents flowing in the frontal zone. It is also possible that the resultant distribution of the thermal and hydrodynamic parameters is affected by the magnetic field enhanced in the inner regions of the wave; in this case the magnetic pressure must be taken into consideration in the computations.

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ASYMPTOTIC ANALYSIS OF THE IGNITION OF A GAS BY A HEATED SURFACE

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The problem of the ignition of a homogeneous hot mixture is a classical problem of combustion theory. Along with the practical significance, its analysis offers the possibility of working out approximate analytic and numerical methods of solution, using one of the simplest problems of nonsteady combustion as an example. The problem of the ignition of a condensed medium was first discussed in [1]. Gas ignition has been discussed numerically in a number of papers (for example, [2, 3] and the review [4]). Recently, efforts have been made to construct approximate analytic solutions of problems concerning ignition on the basis of the method of spliced asymptotic expansions. With the help of these methods an analysis has been carried out in [5, 6] of the ignition of a condensed phase by a luminous flux. The ignition of a condensed phase by a heated surface has been investigated in [7] by one of the authors.*

*V. S. Berman, "Some problems of the theory of the propagation of a zone with exothermic chemical reactions in gaseous and condensed media," Candidate's Dissertation, Institute of Applied Mechanics, Academy of Sciences of the USSR, Moscow (1974).

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